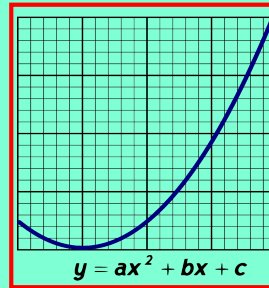


Math 125
Spring 2021
Lecture 22



Simplify $6x - 8x = -2x$

$$1) \quad 6\sqrt{3} - 8\sqrt{3}$$

$$= \boxed{-2\sqrt{3}}$$

$$3) \quad \sqrt{32x} - \sqrt{18x}$$

$$= \sqrt{16}\sqrt{2x} - \sqrt{9}\sqrt{2x}$$

$$= 4\sqrt{2x} - 3\sqrt{2x}$$

$$= 1\sqrt{2x} = \boxed{\sqrt{2x}}$$

$$18x + 16x = 34x$$

$$2) \quad 6\sqrt{18} + 4\sqrt{32}$$

$$= 6\sqrt{9}\sqrt{2} + 4\sqrt{16}\sqrt{2}$$

$$= 18\sqrt{2} + 16\sqrt{2} = \boxed{34\sqrt{2}}$$

$$4) \quad -5\sqrt{75x^3} - 4x\sqrt{27x}$$

$$= -5\sqrt{25x^2}\sqrt{3x} - 4x\sqrt{9}\sqrt{3x}$$

$$= -5 \cdot 5x\sqrt{3x} - 4x \cdot 3\sqrt{3x}$$

$$= -25x\sqrt{3x} - 12x\sqrt{3x}$$

$$= \boxed{-37x\sqrt{3x}}$$

Rationalize the deno.:

$$1) \frac{9}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{9\sqrt{6}}{\sqrt{36}}$$

$$= \frac{9\sqrt{6}}{6} = \frac{3\sqrt{6}}{2}$$

$$2) \frac{-12}{\sqrt{8}} = \frac{-12}{\sqrt{4}\sqrt{2}}$$

$$= \frac{-12}{2\sqrt{2}} = \frac{-6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-6\sqrt{2}}{2}$$

$$3) \sqrt{\frac{27}{5}}$$

$$= \frac{\sqrt{27}}{\sqrt{5}} = \frac{\sqrt{9}\sqrt{3}}{\sqrt{5}} = \frac{3\sqrt{3} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}}$$

$$= \frac{3\sqrt{15}}{\sqrt{25}} = \frac{3\sqrt{15}}{5}$$

$$4) \frac{\sqrt{2}}{\sqrt{2}-1} = \frac{\sqrt{2}}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$= \frac{\sqrt{4} + \sqrt{2}}{\sqrt{4} + \sqrt{2} - \sqrt{2} - 1}$$

$$= \frac{2 + \sqrt{2}}{2 - 1} = \frac{2 + \sqrt{2}}{1} = 2 + \sqrt{2}$$

Simplify

$$1) (4\sqrt{5} - 2)(2\sqrt{5} - 4) = 8\sqrt{25} - 16\sqrt{5} - 4\sqrt{5} + 8$$

$$= 40 - 20\sqrt{5} + 8 = 48 - 20\sqrt{5}$$

$$2) (2\sqrt{3} - 2)^2$$

$$= (2\sqrt{3} - 2)(2\sqrt{3} - 2)$$

$$= 4\sqrt{9} - 4\sqrt{3} - 4\sqrt{3} + 4 = 12 - 8\sqrt{3} + 4 = 16 - 8\sqrt{3} = 8(2 - \sqrt{3})$$

$$3) (\sqrt{15} - \sqrt{10})(\sqrt{15} + \sqrt{10})$$

Conjugates

$$= (\sqrt{15})^2 + \sqrt{150} - \sqrt{150} - (\sqrt{10})^2$$

$$= 15 - 10 = 5$$

Rationalize the deno.:

$$\begin{aligned}
 1) \quad & \frac{\sqrt{2} + 3}{\sqrt{2} - 1} \\
 &= \frac{\sqrt{2} + 3}{\sqrt{2} - 1} \cdot \frac{\sqrt{2} + 1}{\sqrt{2} + 1} \\
 &= \frac{\sqrt{4} + \sqrt{2} + 3\sqrt{2} + 3}{\sqrt{4} + \sqrt{2} - \sqrt{2} - 1} \\
 &= \frac{5 + 4\sqrt{2}}{1} = \boxed{5 + 4\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \frac{\sqrt{18}}{\sqrt{6} + \sqrt{2}} \\
 &= \frac{\sqrt{18}(\sqrt{6} - \sqrt{2})}{(\sqrt{6} + \sqrt{2})(\sqrt{6} - \sqrt{2})} \\
 &= \frac{\sqrt{108} - \sqrt{36}}{\sqrt{36} - \sqrt{12} + \sqrt{12} - \sqrt{4}} \\
 &= \frac{\sqrt{36}\sqrt{3} - 6}{6 - 2} = \frac{6\sqrt{3} - 6}{4} \\
 &= \frac{\overset{3}{\cancel{6}}\sqrt{3} - \overset{3}{\cancel{6}}}{\overset{2}{\cancel{4}} - \overset{2}{\cancel{4}}} \\
 &= \boxed{\frac{3\sqrt{3}}{2} - \frac{3}{2}}
 \end{aligned}$$

Solve

$$\begin{aligned}
 1) \quad & \sqrt{x+1} - 1 = 2 \rightarrow x+1=9 \\
 & \sqrt{x+1} = 3 \quad \checkmark x=8 \\
 & (\sqrt{x+1})^2 = 3^2 \quad \{8\}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & 3\sqrt{x} - \sqrt{x+8} = 0 \\
 & 3\sqrt{x} = \sqrt{x+8} \\
 & (3\sqrt{x})^2 = (\sqrt{x+8})^2 \\
 & 9x = x+8 \\
 & 9x - x = 8 \\
 & 8x = 8 \\
 & \checkmark x=1
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & x = \sqrt{x^2 + 5x + 10} \\
 & x^2 = (\sqrt{x^2 + 5x + 10})^2 \\
 & \cancel{x^2} = \cancel{x^2} + 5x + 10
 \end{aligned}$$

$$\begin{aligned}
 \text{check} \quad & -5x = 10 \quad x = -2 \quad \text{E.S.} \\
 & x = \sqrt{x^2 + 5x + 10} \rightarrow -2 = \sqrt{4} \\
 & -2 = \sqrt{(-2)^2 + 5(-2) + 10} \rightarrow -2 = 2 \\
 & -2 = \sqrt{4 - 10 + 10} \rightarrow \text{False}
 \end{aligned}$$

\emptyset

$\{ \}$

Solve $\sqrt{2x-3} + 3 = x$

$\sqrt{2x-3} = x-3$

$(\sqrt{2x-3})^2 = (x-3)^2 \rightarrow (x-3)(x-3)$

$2x-3 = x^2 - 6x + 9$

$0 = x^2 - 6x + 9 - 2x + 3$

$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

Check $x=6$ ✓ $x-6=0$ $x-2=0$

$\sqrt{2x-3} + 3 = x$ $\sqrt{x-6}$ ~~$x=2$~~ E.S.

$\sqrt{2(6)-3} + 3 = 6$

$\sqrt{9} + 3 = 6$

$3 + 3 = 6$

$6 = 6$ ✓

Check $x=2$

$\sqrt{2(2)-3} + 3 = 2$

$\sqrt{1} + 3 = 2$

$1 + 3 = 2$ **False**

$\{6\}$

Solve $\sqrt{x+21} - \sqrt{x} = 3$

Hint: Isolate one radical

$\sqrt{x+21} = \sqrt{x} + 3$

$(\sqrt{x+21})^2 = (\sqrt{x} + 3)^2$

$x+21 = (\sqrt{x} + 3)(\sqrt{x} + 3)$

$(x) + 21 = (x) + 3\sqrt{x} + 3\sqrt{x} + 9$

$21 - 9 = 6\sqrt{x}$

$12 = 6\sqrt{x}$

$2 = \sqrt{x}$

$(\sqrt{x})^2 = 2^2$

$\checkmark x=4$ checks

$\{4\}$

$\sqrt{x}\sqrt{x} = (\sqrt{x})^2 = x$

Divide:

$$1) \frac{-3i}{1+2i}$$

$$= \frac{-3i(1-2i)}{(1+2i)(1-2i)}$$

$$= \frac{-3i + 6i^2}{1 - 2i + 2i - 4i^2}$$

$$= \frac{-3i - 6}{1 - 4(-1)} = \frac{-3i - 6}{5}$$

$$= \frac{-6 - 3i}{5}$$

$$= \boxed{\frac{-6}{5} - \frac{3}{5}i}$$

$$2) \frac{1+i}{1-i}$$

$$= \frac{(1+i)(1+i)}{(1-i)(1+i)}$$

$$= \frac{1+i+i+i^2}{1+i-i-i^2}$$

$$= \frac{2i}{1-(-1)}$$

$$= \frac{2i}{2} = \boxed{i}$$

Simplify:

$$1) i^{180}$$

$$= (i^2)^{90}$$

$$= (-1)^{90} = \boxed{1}$$

$$2) i^{111}$$

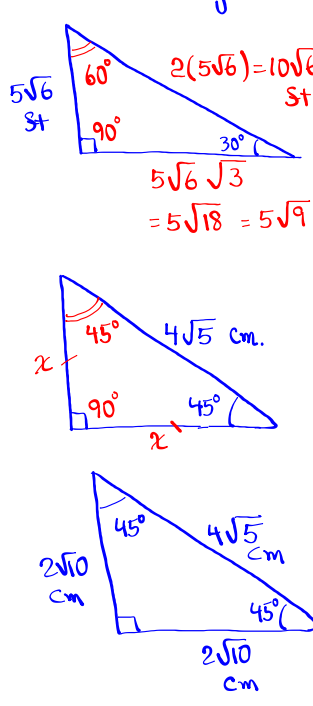
$$= i^{110} \cdot i$$

$$= (i^2)^{55} \cdot i$$

$$= (-1)^{55} i$$

$$= -1i = \boxed{-i}$$

Find the missing sides/angles:



$5\sqrt{6}$ St
 60°
 90°
 30°
 $2(5\sqrt{6}) = 10\sqrt{6}$ St
 $5\sqrt{6}\sqrt{3}$
 $= 5\sqrt{18} = 5\sqrt{9\sqrt{2}} = 5 \cdot 3\sqrt{2} = 15\sqrt{2}$ St

45°
 90°
 45°
 $4\sqrt{5}$ cm
 x
 x

Using Pythagorean Thrm

$$x^2 + x^2 = (4\sqrt{5})^2$$

$$2x^2 = 16 \cdot 5$$

$$x^2 = 8 \cdot 5$$

$$x^2 = 40$$

$$x = \sqrt{40}$$

$$x = \sqrt{4} \sqrt{10}$$

$$x = 2\sqrt{10}$$

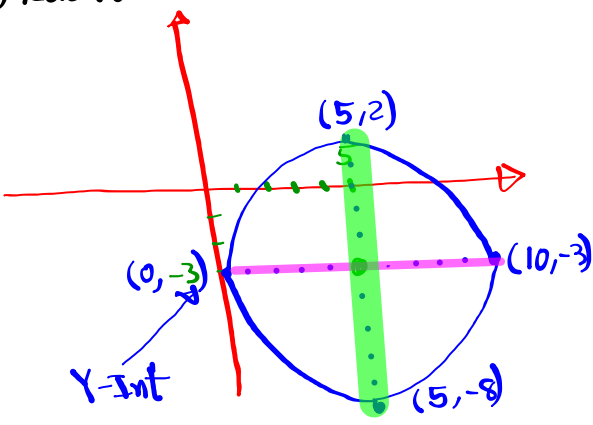
45°
 90°
 45°
 $2\sqrt{10}$ cm
 $4\sqrt{5}$ cm
 $2\sqrt{10}$ cm

Consider $(x - 5)^2 + (y + 3)^2 = 25$

1) Center $(5, -3)$ 2) Radius $r = 5$ 3) Draw

4) Domain $[0, 10]$

5) Range $[-8, 2]$



Consider $\frac{(x+3)^2}{9} + \frac{(y-4)^2}{4} = 1$

a) Center $(-3, 4)$ b) $a^2 = 9$ c) $b^2 = 4$

$a = 3$ $b = 2$

d) Draw

5) Discuss domain & Range Domain: $[-6, 0]$
Range: $[2, 6]$

6) Discuss intercepts x-Int: None y-Int: $(0, 4)$

Consider $36x^2 + 9y^2 = 324$

1) Write in $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ Hint: Make RHS=1

Divide by 324 $\frac{36x^2}{324} + \frac{9y^2}{324} = \frac{324}{324} \Rightarrow \frac{x^2}{9} + \frac{y^2}{36} = 1$

2) Center $(0, 0)$ 3) $a = 3$ 4) $b = 6$

5) Draw

6) Discuss domain & Range.
Domain: $[-3, 3]$ Range: $[-6, 6]$

7) Discuss All intercepts. x-Ints: $(-3, 0), (3, 0)$
y-Ints: $(0, 6), (0, -6)$

Draw $4(x-3)^2 + 25(y+2)^2 = 100$

Divide by 100

$$\frac{(x-3)^2}{25} + \frac{(y+2)^2}{4} = 1$$

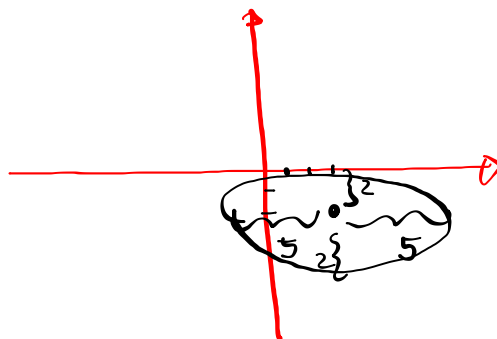
Center (3, -2)

a=5 b=2

Circle ✓

Ellipse ✓

Hyperbola ??

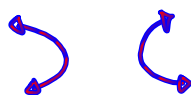


Hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

OR $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

Center (0,0)

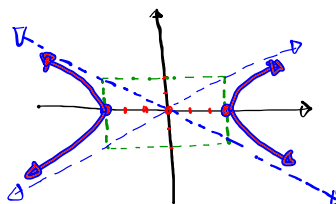


$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

Center (0,0)

a=3, b=2

open sideways



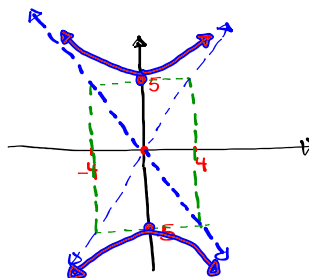
$$\frac{y^2}{25} - \frac{x^2}{16} = 1$$

Center (0,0)

a^2=16 a=4

b^2=25 b=5

opens up/down



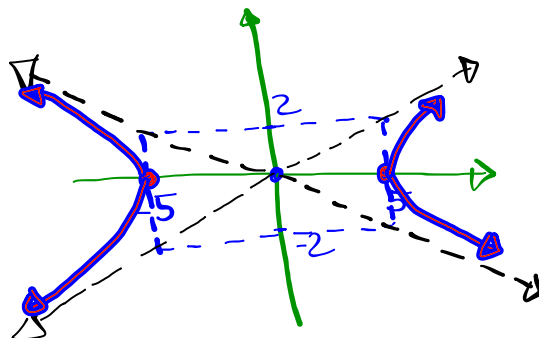
$$4x^2 - 25y^2 = 100$$

↑ Hyperbola ↑ has to be 1

Divide by 100 $\Rightarrow \frac{4x^2}{100} - \frac{25y^2}{100} = \frac{100}{100}$

$$\frac{x^2}{25} - \frac{y^2}{4} = 1$$

open sideways



Draw $4x^2 - 16y^2 = -64$

Divide by -64

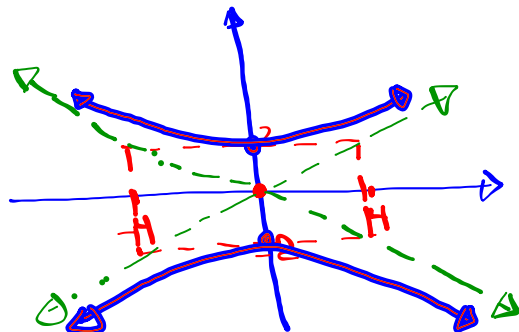
$$\frac{4x^2}{-64} - \frac{16y^2}{-64} = \frac{-64}{-64} \Rightarrow -\frac{x^2}{16} + \frac{y^2}{4} = 1$$

Rewrite this $\Rightarrow \frac{y^2}{4} - \frac{x^2}{16} = 1$

Center (0,0) $a^2=16$
 $a=4$

$$b^2=4 \quad b=2$$

opens up/down



Class QZ 18

Solve & Check: $\sqrt{4x+1} + 1 = x$